COMBUSTION STABILITY OF A DISPERSED FUEL IN A COMBUSTION CHAMBER

S. K. Aslanov

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A combustion instability criterion and an analogy with the burning of a solid propellant element are established as a result of an analysis of the behavior of the gasdynamic disturbances superimposed on the process of turbulent combustion of an atomized fuel in a one-dimensional combustion chamber.

We will consider the schematized combustion chamber of constant cross section shown for the undisturbed state in the figure at a. Oxidizer and atomized fuel are introduced into region 1 ($-L \leq x \leq 0$) through the end AA'. A certain volume is required for the evaporation and mixing of the fuel with the oxidizer (for example, air), before ignition occurs at BB'. Consequently, liquid particles will exist up to the flame itself. The thoroughly mixed fuel mixture burns in turbulent combustion zone 3 ($0 \leqslant x \leq L_m$) and the combustion products enter zone 2 ($x \geq L_m$). Thus, this one-dimensional model of the combustion process in the chamber has three regions of flow under conditions of homogeneous turbulence, whose averaged parameters p, ρ, v, c, S, $\varkappa$, c_p are denoted by corresponding numerical subscripts.

If turbulence plays a decisive part in the combustion mechanism and the flame propagation velocity u_m (which in our model coincides with v_1) and the width of the flame zone L_m are determined, as in the case of gas combustion, chiefly by the mean fluctuation velocity v', then, following K. I. Shchelkin [1] (§14), we can write approximately

$$u_m = u_n + Kv', \quad L_m = Al\,(v'/u_n)^m, \quad 0.5 \leqslant m \leqslant 1, \qquad (1)$$

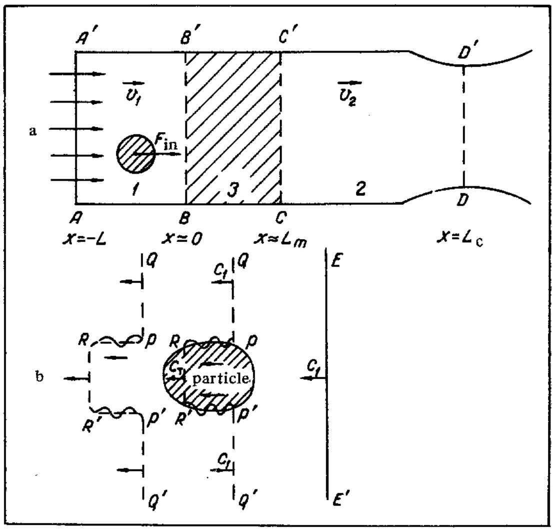
where A and K are certain constants of the order of unity. Otherwise it is necessary to use other more complex expressions reflecting the actual relations between the vaporization and combustion processes.

If the burning velocity changes slightly as a result of certain random influences of an internal character, zone 3 will experience a slight displacement relative to the undisturbed state, which can be expressed in the form

$$\varepsilon = A_0 \exp \omega t. \qquad (2)$$

Being displaced as a unit, the flame becomes a source of acoustic disturbances (p_j', v_j') in the starting mixture in region 1 (j = 1) and in the combustion products in region 2 (j = 2) as well as of an entropy disturbance S_2' carried downstream by the flow. These disturbances are found by analogy with [2] from the solution of the linearized gasdynamic equations and take the form

$$v_j' = v_{j1}' + v_{j2}',$$



Schematized combustion chamber (a) and dispersion of acoustic front on atomized fuel particles (b).

$$\frac{p_j'}{\rho_j v_j} = \beta_{j1} v_{j1}' + \beta_{j2} v_{j2}', \quad \beta_{jc} = -\left(1 + \frac{\omega}{v_j \gamma_{jc}}\right),$$

$$S_j' = (j-1) A_4 \exp\left[-\frac{\omega}{v_j}(x - L_m) + \omega t\right],$$

$$\gamma_{jc} = -\frac{\omega}{v_j}\,\frac{M_j}{M_j + (-1)^c},$$

$$v_{jc}' = A_{jc} \exp\{\gamma_{jc}[x + (1-j)L_m] + \omega t\},$$

$$M = v/c < 1. \qquad (3)$$

On the other hand, the internally induced disturbances also react on the combustion process as a result of the interaction of the acoustic waves and the internal structure of regions 1 and 3. This may be expected to produce changes in the burning velocity δu_m, i.e., the propagation velocity of the flame relative to the disturbed starting mixture directly ahead of the boundary of the turbulent combustion zone. In the linear formulation this change can be written as

$$\delta u_m = v_1'|_{x=0} - \frac{d\varepsilon}{dt} = \delta u_m^{(1)} + \delta u_m^{(3)}, \qquad (4)$$

where the superscripts denote the number of the region (figure, a) in which the increment is generated. The increase in burning velocity due to the mechanism of interaction of the acoustic waves and the turbulent structure within flame zone 3, assuming, of course, that this structure determines the combustion processes, was found in [2] in the form

$$\delta u_m^{(3)} = D_3 \frac{1}{L_m} \int_0^{L_m} v_3' \, dx,$$

$$D_3 = C \frac{L_m}{L_c} = C \frac{l}{L_c} \left(\frac{v'}{u_n} \right)^m. \qquad (5)$$

The constant C, which is not very different from unity and is subject to experimental determination, includes all the effects not taken into account in the calculation scheme adopted for region 3. The acoustic disturbance within the flame v_3' is determined from (3) for j = 3 and can be expressed in terms of the acoustic waves of regions 1 and 2. In fact, the acoustic disturbance originating in the vicinity of BB' (figure, a) is propagated in the form of diverging waves: v_{11}' into region 1 and v_{32}' into region 3. Since the burning velocity v_1 is much less than the speed of sound, from the acoustic standpoint (correct to M_1^2) it is possible to disregard the flux across the leading edge of the flame BB'. Then in the vicinity $x \simeq 0$ the acoustic velocities should coincide: $v_{11}' = v_{32}'$. A similar condition can be introduced in the vicinity of CC' ($x \simeq L_m$) $v_{31}' = v_{22}'$ for the waves diverging into regions 3 and 2. The substitution of solutions (3) into these conditions gives $A_{11} = A_{32}$, $A_{31} = A_{22}$.

The second feedback mechanism generating $\delta u_m^{(1)}$ consists in the mechanical interaction of the acoustic disturbances propagating through the gas phase of region 1 and the droplets (particles) of atomized fuel, one of which is represented schematically in the figure to a magnified scale. In passing through gas region 1, the sound waves generate an acoustic acceleration dv_1'/dt, and the elements of the gas mixture are subjected to a force proportional to that acceleration. Therefore the particles of dispersed fuel (in the liquid or solid phase) will experience the effect of an opposite force relative to the surrounding gas, in accordance with Newton's third law of motion. In other words, in the disturbed state each fuel particle of mass m in region 1 will be acted upon relative to the gas by an inertia force $F_{in} = -m(dv_1'/dt)$, where $m \sim \rho_f d$. As a result of the operation of these inertia forces throughout region 1, i.e., $\int_{-L}^{0} F_{in} \, dx$, additional fuel particles, as compared with the undisturbed state of the process, will be delivered to flame zone 3. Their combustion leads immediately to an increase in burning velocity $\delta u_m^{(1)}$. Consequently, for the given feedback mechanism the specific work done by the inertia forces will serve as a measure of the change of burning velocity, so that we can write

$$\delta u_m^{(1)} = - D_1 \int_{-L}^{0} \frac{dv_1'}{dt} \, dx =$$

$$= - D_1 c_1 M_1 \int_{-L}^{0} \left(\frac{1}{v_1} \frac{\partial v_1'}{\partial t} + \frac{dv_1'}{\partial x} \right) dx. \qquad (6)$$

The constant D_1 has the dimension of reciprocal velocity and characterizes the intensity of the feedback described. Strictly speaking, in region 1 the acoustic waves are propagated in a two-phase medium. Therefore the picture of the passage of the acoustic front through a fuel particle is only very schematically represented by the model shown in the figure at b. The difference between the speed of sound c_1 in the gas phase and the longitudinal compression waves in the fuel $c_f > c_1$ will result in the dispersion of the incident acoustic front EE'. Whereas in the gas phase the front occupies the position QPP'Q', in the liquid (solid) phase it is able to propagate to RR', imparting acoustic velocities to the medium in the region RPP'R'. Hence at the fuel-gas boundaries RP (R'P') we get tangential velocity discontinuities that are unstable [4] and subsequently become turbulent, producing dissipative energy losses and hence weakening the feedback effect. Naturally, this weakening effect on the feedback increases with increase in the difference between c_f and c_1, so that we can write $D_1 c_1 = D_1' c_1/c_f$ for the dimensionless quantity D_1'. The work done by the inertia forces F_{in} is proportional to the masses of the atomized fuel particles m. Hence, in dimensionless form, referred to the gaseous mass of region 1, we have $D_1' = D_1'' \rho_f d/\rho_g L$. It should be noted that the fuel particles in region 1 are of different sizes (mass). However, in the case of a liquid medium the process will be complicated by the simultaneous evaporation of the droplets, i.e., in general the mass of an individual particle of liquid fuel will be variable, decreasing in proportion to the evaporation rate. Therefore the dimension (diameter) of the particles d, appearing in the expression for D_1', must naturally be understood as a certain mean over the entire suspension in region 1, i.e., the ratio of the entire dispersed mass of fuel to the product of the number of particles and the fuel density ρ_f. The effect of the change of droplet mass due to evaporation as it moves through region 1 will reduce the inertia force F_{in}, thus weakening the intensity of the feedback. Hence, to take this process into account, the coefficient D_1'' must have an inverse dependence on the rate of evaporation of the fuel droplets or in the simplest case may be taken inversely proportional to that rate. Furthermore, the rate of increase of burning velocity (6) will be determined not only by the additional mass of fuel entering the flame zone owing to the disturbances but also, of course, by its degree of readiness for combustion. The latter is determined by the heating efficiency of the preheating zone, the extent of which is controlled by the thermal diffusivity of the fuel χ_f. Consequently, we must also have $D_1'' \sim \chi_f/\chi_g$. Thus, we finally obtain

$$D_1' \frac{c_1}{c_f} = B \frac{c_1}{c_f} \frac{\chi_f}{\chi_g} \frac{\rho_f}{\rho_g} = B \frac{c_1}{c_f} \frac{\lambda_f}{\lambda_g} \frac{c_{pg}}{c_{pf}},$$

$$B = B_1 \frac{d}{L} \qquad (7)$$

where the constant B, on the order of unity and subject to experimental determination, comprises all the effects not taken into account in the calculation scheme adopted. In particular, for the case of liquid fuel the coefficient B (more accurately, B_1), as pointed out above, has an inverse dependence on the rate of evaporation of the fuel droplets. Thus, the two constant multipliers B and C, involved in the description of the feedback mechanism (5)–(7), by virtue of their experimental nature establish a correspondence between the proposed theory and actual applications.

Feedback equation (4) is now written in the form

$$v_1'|_{x=0} - \frac{d\varepsilon}{dt} = D_3 \frac{1}{L_m} \int_0^{L_m} v_3' \, dx -$$

$$- D_1' \frac{c_1}{c_f} M_1 \int_{-L}^{0} \left(\frac{1}{v_1} \frac{\partial v_1'}{\partial t} + \frac{d v_1'}{\partial x} \right) dx. \tag{8}$$

As the boundary condition at the end of the combustion chamber AA' we can use the condition

$$v_1' = 0 \quad \text{at} \quad x = -L. \tag{9}$$

The boundary condition at the chamber outlet DD' will depend on the physical conditions of flow through the nozzle. In the case of completely developed Jouguet deflagration, owing to the sonic speed of the combustion products the disturbances cannot return to the flame from region 2, so that it is necessary to take into account only the departing sound waves and $A_{21} = 0$. In other cases there will be a partial return of the disturbances to the flame owing to reflections in the convergent part of the nozzle. Considering the one-dimensionality of the model and the large losses in the combustion products, we will disregard this effect.

As in [2], to correlate the disturbed states of the averaged parameters of regions 1 and 2 we employ the laws of continuity of the mass, momentum, and energy fluxes assuming that the mixture burns up completely in the flame 3:

$$\alpha \left[v_1' - \frac{d\varepsilon}{dt} + \frac{p_1'}{\rho_1 v_1} M_1^2 \right]_{x=0} =$$

$$= \left[v_2' - \frac{d\varepsilon}{dt} + \frac{p_2'}{\rho_2 v_2} M_2^2 - \frac{v_2}{c_{p_2}} S_2' \right]_{x=L_m},$$

$$\left[\frac{p_1'}{\rho_1 v_1} (1 + M_1^2) + 2 v_1' \right]_{x=0} =$$

$$= \left[\frac{p_2'}{\rho_2 v_2} (1 + M_2^2) + 2 v_2' - \frac{v_2}{c_{p_2}} S_2' \right]_{x=L_m},$$

$$\frac{1}{\alpha} \left[v_1' - \frac{d\varepsilon}{dt} + \frac{p_1'}{\rho_1 v_1} \right]_{x=0} =$$

$$= \left[v_2' - \frac{d\varepsilon}{dt} + \frac{p_2'}{\rho_2 v_2} + \frac{v_2}{c_{p_2}} \frac{S_2'}{(\varkappa_2 - 1) M_2^2} \right]_{x=L_m},$$

$$\alpha = \frac{v_2}{v_1} = \frac{\rho_1}{\rho_2} > 1. \tag{10}$$

Since we have disregarded the turbulent dissipation effect and hence its stabilizing influence on the disturbances, the instability criterion should be strengthened somewhat. This is not important, since the effect in question can be included in the constants B and C, which enter into the feedback equation (8) and are still subject to experimental determination. Substitution of (2), (3) into (8)–(10) gives a linear system for the constants A_0, A_{11}, A_{12}, A_4 with a characteristic equation for the eigenvalue ω:

$$f(\omega) = 0,$$

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$$= \sum_{c=1}^{2} (-1)^{c-1} \exp \gamma_{1c} L \left(\Phi_c + \frac{\alpha - 1}{1 + M_2} M_2 b_1 F_c \right) -$$

$$- a(\alpha - 1) \left[\frac{\alpha - 1}{\alpha} + \frac{1 + M_2}{(\varkappa_2 - 1) M_2^2} \right],$$

$$b_c = \frac{D_3}{\gamma_{3c} L_m} \{ 1 - \exp [(-1)^c \gamma_{3c} L_m] \},$$

$$a = (b_2 + 1) \exp \gamma_{11} L -$$

$$- \exp \gamma_{12} L + D_1' \frac{c_1}{c_f} (\exp \gamma_{11} L + \exp \gamma_{12} L - 2),$$

$$F_c = f_c \left[\frac{\alpha - 1}{\alpha} + \frac{(-1)^c}{M_1} \frac{1 + (-1)^c M_1}{(\varkappa_2 - 1) M_2^2} \right],$$

$$f_c = 1 + (-1)^c M_1,$$

$$\Phi_c = f_c \left\{ (\alpha - 1) \left[1 - \frac{(-1)^c}{\alpha M_1} \right] + \right.$$

$$\left. + \frac{1}{(\varkappa_2 - 1) M_2^2} \left[\frac{(-1)^{c-1}}{M_1} - 1 + \alpha \frac{1 + M_2}{M_2} \right] \right\}. \tag{11}$$

Since as $\omega \to +\infty$, $b_k = 0$, $a = \exp \gamma_{11} L [1 + D_1'(c_1/c_f)]$, while at $\omega = 0$, $\gamma_{jk} = 0$, $b_k = (-1)^{k-1} D_3$, $a = b_2$, we see that $f(0)$ and $f(+\infty)$ will have different signs if the following inequalities are simultaneously satisfied (correct to M_1^2):

$$R_1 < 1 < R_2 \quad \text{or} \quad R_1 > 1 > R_2$$

$$R_1 = D_1' \frac{c_1}{c_f} M_1 (\alpha - 1) \psi_1,$$

$$\psi_1 = \frac{q + \dfrac{1}{M_2}}{(1 - M_1) \left[(1 + M_1) q + M_1 \left(\dfrac{1}{M_2} - 1 \right) \right]},$$

$$R_2 = D_3 (\alpha - 1) \psi_2,$$

$$\psi_2 = \frac{1 - \dfrac{M_1}{2} (1 + M_2) \left(q + \dfrac{1}{M_2} \right)}{(1 + M_2) q},$$

$$q = 1 + \frac{\alpha - 1}{\alpha} (\varkappa_2 - 1) M_2. \tag{12}$$

The latter, which ensure the existence of at least one positive root ω of Eq. (11), are sufficient criteria of the instability of the combustion process in our model of a combustion chamber with disturbances increasing exponentially in time. In particular, it follows from (12) that, if $R_1 < 1$, i.e., the dispersity of the medium in region 1 is not large, then the flame turbulence mechanism is sufficient for the development of instability if

$$C \frac{L_m}{L_c} (\alpha - 1) \psi_2 > 1.$$

However, if $R_2 < 1$, i.e., the turbulence in the flame zone is not great or does not play a decisive role in the combustion processes, then the fuel-oxidizer mixture dispersity mechanism is sufficient for instability if

$$B \frac{c_1}{c_f} \frac{\chi_f}{\chi_g} \frac{\rho_f}{\rho_g} M_1 (\alpha - 1) \psi_1 > 1, \quad B = B_1 \frac{d}{L},$$

which in the case of weak deflagration (small Mach numbers) takes the form

$$B \frac{c_1}{c_f} \frac{\chi_f}{\chi_g} \frac{\rho_f}{\rho_g} \frac{\alpha - 1}{1 + M_2/M_1} > 1. \tag{13}$$

In conclusion, we will examine the gasdynamic analogy between the process of combustion of an atomized fuel in a combustion chamber (when the turbulence is not important and the Mach numbers are small) and the process of combustion of a solid propellant element [5]. In fact, the role of the rigid chamber end AA', reflecting the acoustic disturbances, is now played by the solid combustion surface. The fuel mixture, formed as a result of fusion and gasification of the material in a thin surface layer of fuel, burns in the flame front, which is separated from the solid surface by a "dark" induction zone, analogous to region 1 (figure, a). A feedback is again created by the fact that in the disturbed state, as a result of the total work done by the inertia forces in the "dark" zone on the particles dispersed in it, additional fine droplets and particles—an air suspension of the fuel—enter the flame zone [5], thereby increasing the burning rate. Accordingly, instability criterion (13) will also serve for the process of combustion of a solid propellant element (grain). This conclusion is best confirmed by the exact correspondence between condition (13) with B = 1 and the criterion for the amplification of high-frequency vibrations in a burning solid propellent obtained in [5]. In fact, according to [1], $\alpha - 1 = (\varkappa - 1)Q/c_1^2$, if it is assumed that $\varkappa_1 = \varkappa_2 = \varkappa$ and M_1^2 is neglected. In the same approximation the present author [2] found $M_2/M_1 \simeq \sqrt{\alpha}$ and hence $c_2/c_1 = \sqrt{\alpha}$. Consequently, condition (13) is transformed into inequality (18) of [5].

NOTATION

L is the distance of the flame from end of chamber; L_m is the width of the turbulent flame; l is the scale of turbulence; v' is the mean turbulent fluctuation velocity; u_n is the normal burning velocity; u_m is the turbulent burning velocity; δ is the increment symbol; ε is the small displacement of flame; p is the pressure; ρ is the density; v is the flow velocity; c is the speed of sound; S is the entropy; c_p is the specific heat; $\varkappa$ is the ratio of specific heats; ω is the eigenvalue; subscripts 1, 2, and 3 apply to the regions in front of and behind the flame and the flame zone, respectively; a prime denotes the corresponding perturbations; M is the Mach number; L_c is the length of the combustion chamber; m is the mean mass of atomized fuel particle; d is the mean characteristic dimension (diameter) of particle, the subscripts f and g correspond to the fuel and the gas, respectively; χ is the thermal diffusivity; λ is the thermal conductivity; $\alpha = v_2/v_1$; Q is the reaction energy per unit mass of mixture.

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